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Abstract

A number of multiproduct firms participate in the markets for two goods, competing to a number of single–product firms in each market. Although the joint production of cost substitutes is inefficient, participation in both markets yields, under a broad range of conditions, higher profits than participation in only one of them. Inefficient multiproduct firms may be contained in a *long–run equilibrium configuration*. If such configurations contain *integer* number of firms — instead of *fractions* of them — inefficient industry configurations are more likely to be sustained in the long run.

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1 Introduction

In this paper I study the properties of equilibrium in a multiproduct industry which supplies the markets for two goods which are not related in demand.[1] The two goods may be complements or substitutes in production. *Cost complementarity* and *weak subadditivity* of fixed costs of a multiproduct plant, as compared to the sum of fixed costs of the *'stand alone'* production of the two products, lead to *global economies of scope. Cost substitutability* and *weak subadditivity* of fixed costs of a multiproduct plant as compared to the sum of fixed costs of the *'stand alone'* production of the two products lead to *local economies of scope.* I study short–run equilibrium under the assumption that the industry consists of three types of firms: firms producing both products, firms producing only the first product and firms producing only the second product.

Multiproduct firms possess the *know how* for the production of both products, while **single product firms** possess the *know how* for the production of only one of them. I assume that this asymmetry is a result of exogenous factors.[2]

Multiproduct firms may drop one of their products and become single–product, if they find it profitable to do so. In my model, the decision of a multiproduct firm to become single–product is equivalent to the exit of a multiproduct firm followed by the entry of a single–product one. Therefore, multiproduct firms are treated as *potentially multiproduct* ones. This assumption corresponds to the first stage of a two–stage game in which *potentially multiproduct* firms first choose whether to enter as multiproduct or single–product ones and then make their quantity–setting decision.[3]

While multiproduct firms are implicitly allowed to choose between multiproduct and single-product activity, single–product firms are bound to produce

[1]The products are sold in two independent markets. The two markets are separate in the sense that the two goods are neither substitutes nor complements in consumer preferences.

[2]In the real world similar situations result from limited availability of licensing agreements concerning products whose production technology is protected by a patent which belongs to R&D-oriented firms. Kim, Röller and Tombak (1992) assume that two competitors have the possibility of choosing between two types of technologies. A *flexible technology* can be used in the production of both goods, while a *dedicated technology* can be used in the production of only one of the two goods. They show that in the presence of demand substitutability the two firms will choose the same type of technology. Therefore, a mixed industry structure (that is an industry in which multiproduct and single–product firms co–exist) cannot be an equilibrium under demand substitutability.

[3]This feature of the model was pointed out to me by Professor Gabszewicz.

only one of the two products. I assume that firms undertake production in a single plant.

In this analytical framework I reach the following main results:

In the presence of cost complementarity, a multiproduct firm contributes more to the total output of each of the two products than any of its single–product rivals. In the presence of cost complementarity the opposite holds. If the two products are cost complements and the fixed costs of a multiproduct firm are equal to the sum of the fixed costs of *'stand alone'* production, the profits of a multiproduct firm from each product are higher than the profits of a single–product manufacturer of the same product. If the two products are cost substitutes, the opposite holds.

However, under a broad range of values of the parameters, the total profits of a multiproduct firm are higher than the profits of a single–product manufacturer of any type. When the fixed costs of joint production are lower than the sum of the fixed costs of *'stand alone'* production of the two goods, the range of conditions under which multiproduct activity is preferred to single–product activity becomes broader. This property of the short–run equilibrium and the assumption of limited production possibilities of single–product firms results in multiple long–run equilibria that contain multiproduct and single–product firms.

In Section 3, I extend the definition of the *long–run equilibrium number of firms* to its multiproduct analogous, the *long–run equilibrium industry configuration.* Firms enter until the profits of each type of firm are driven to zero. I treat the *integer problem* in two ways. First, I allow for *fractions* of firms to enter into the industry. In general, the long–run equilibrium industry configuration will exclusively contain either multiproduct or single–product firms. Multiple mixed equilibria require that economies or diseconomies of scope reflected in the fixed costs of production co–exist with diseconomies or economies of scope which affect unit costs of production. Under the assumption that industry configurations contain integer numbers of firms, there is a broader range of conditions that result in multiple mixed long–run equilibria. Inter–market mergers of single–product firms into multi–product plants lead from one long–run equilibrium configuration to another.

2 Multiproduct Industry Structure and Short–Run Equilibrium

A number of identical multiproduct firms compete with each other in a quantity–setting fashion in the supply of two products. At the same time, they have a number of single–product rivals in the supply of each product.

2.1 The model

In the market for product 1 the inverse demand function is given by

$$P_1 = a - Q_1 \tag{1}$$

and in the market for product 2 the inverse demand function is given by

$$P_2 = h - Q_2 \tag{2}$$

with $a > 0$ and $h > 0$. Q_1 and Q_2 are the aggregate outputs of products 1 and 2 supplied in these two markets respectively. Therefore,

$$Q_1 = \Sigma_{k=1}^{\kappa} x_{k1} + \Sigma_{l=1}^{\lambda} y_{l1} \tag{3}$$

and

$$Q_2 = \Sigma_{k=1}^{\kappa} x_{k2} + \Sigma_{m=1}^{\mu} y_{m2} \tag{4}$$

where κ is the number of multiproduct firms, λ the number of single–product firms in market 1 and μ the number of single–product firms in market 2. The *multiproduct industry structure* or *industry configuration* is then $S = (\kappa, \lambda, \mu)$. x_{k1} and x_{k2} are quantities of products 1 and 2 produced by a multiproduct manufacturer k. The output of a single–product manufacturer l in market 1 is y_{l1} and the output of a single–product manufacturer m in market 2 is y_{m2}.

For a single–product manufacturer l in market 1 costs of production are given by

$$C_{l1} = F_1 + b_1 y_{l1} \tag{5}$$

For a single–product manufacturer m in market 2 production costs are

$$C_{m2} = F_2 + b_2 y_{m2} \tag{6}$$

If the two products are produced jointly by a multiproduct manufacturer, production costs are given by

$$C_{k12} = F_{12} + b_1 x_{k1} + b_2 x_{k2} + 2\beta\sqrt{x_{k1} x_{k2}} \tag{7}$$

I assume that $b_1 > 0$, $b_2 > 0$, $F_1 \geq 0$, $F_2 \geq 0$, $F_{12} \geq 0$ and that $F_1 + F_2 \geq F_{12}$ denotes weak subadditivity of fixed costs of joint production as compared to the sum of the fixed costs of the *'stand alone'* production of the two products.

Note that $\partial^2 C_{k12}/(\partial x_{k1} \partial x_{k2}) = \beta/2\sqrt{x_{k1} x_{k2}}$. Therefore, the sign of the parameter β determines whether the two products are cost complements (if $\beta < 0$) or cost substitutes (if $\beta > 0$). If $\beta = 0$ the marginal cost of each good is independent of the level of output of the other. I shall refer to the parameter β

as *the degree of cost complementarity or substitutability.* The multiproduct cost function (7) implies that costs are linear along a constant ray of production. Subadditivity of fixed costs of joint production together with cost complementarity lead to global economies of scope. Subadditivity of fixed costs and cost substitutability lead to local economies of scope.[4]

The profit of a multiproduct firm k is given by

$$\begin{aligned}\Pi_k = (a - \Sigma_{i\neq k}x_{i1} - \Sigma_{l=1}^{\lambda}y_{l1} - x_{k1})x_{k1} \\ +(h - \Sigma_{i\neq k}x_{i2} - \Sigma_{m=1}^{\mu}y_{m2} - x_{k2})x_{k2} \\ -b_1x_{k1} - b_2x_{k2} - 2\beta\sqrt{x_{k1}x_{k2}} - F_{12}\end{aligned} \quad (8)$$

The profit of a single–product firm l which produces product 1 is given by

$$\Pi_l = (a - \Sigma_{k=1}^{\kappa}x_{k1} - \Sigma_{j\neq l}y_{j1} - y_{l1})y_{l1} - b_1y_{l1} - F_1 \quad (9)$$

and the profit of a single–product firm m which produces product 2 is given by

$$\Pi_m = (h - \Sigma_{k=1}^{\kappa}x_{k2} - \Sigma_{n\neq m}y_{n2} - y_{m2})y_{m2} - b_2y_{m2} - F_2 \quad (10)$$

2.2 Equilibrium

I assume that $a - b_1 = h - b_2 = M$ which can be interpreted as equality between the sizes of the two markets.[5] Firms set quantities to maximise their profits, taking their rivals' outputs as given. Maximisation of the profit functions (8), (9) and (10) requires the solution of a system of $(2\kappa + \lambda + \mu)$ equations:

$$\begin{aligned}\partial\Pi_k/\partial x_{k1} = 0 = \\ M - 2x_{k1} - \Sigma_{i\neq k}x_{i1} - \Sigma_{l=1}^{\lambda}y_{l1} - \beta\sqrt{x_{k2}/x_{k1}}\end{aligned} \quad (11)$$

$$\begin{aligned}\partial\Pi_k/\partial x_{k2} = 0 = \\ M - 2x_{k2} - \Sigma_{i\neq k}x_{i2} - \Sigma_{m=1}^{\mu}y_{m2} - \beta\sqrt{x_{k1}/x_{k2}}\end{aligned} \quad (12)$$

$$\partial\Pi_l/\partial y_{l1} = 0 = M - 2y_{l1} - \Sigma_{j\neq l}y_{j1} - \Sigma_{k=1}^{\kappa}x_{k1} \quad (13)$$

$$\partial\Pi_m/\partial y_{m2} = 0 = M - 2y_{m2} - \Sigma_{n\neq m}y_{n2} - \Sigma_{k=1}^{\kappa}x_{k2} \quad (14)$$

Due to non–linearity of the terms $\beta\sqrt{x_{k1}/x_{k2}}$ and $\beta\sqrt{x_{k2}/x_{k1}}$ equations (11) and (12) are non–linear. I assume that multiproduct firms produce quantities of the two products away from the axes and near the ray of production

[4]For a further discussion of the properties of the cost function (7) see Georgantzis (1992).
[5]The term is due to Martin (1993).

$x_{k1} = x_{k2}$. This assumption requires equal numbers of single–product firms in the two markets ($\lambda = \mu = \nu$). Such *symmetric industry configurations* can be described by $S = (\kappa, \nu)$, where ν is the number of single–product firms in each market.[6]

Under the symmetry assumption, the interaction term becomes linear and is expressed in the reaction functions of multiproduct firms as a constant equal to β. Multiproduct firms are identical. The same holds for single–product firms in the same market. I sum the reaction functions of identical firms in the same market. The system reduces to a system of two equations with two unknown variables, which correspond to the output of each type of firm in each market. Then, $x_1 = x_2 = x$ gives the output of each multiproduct firm in each market 1, and $y_1 = y_2 = y$ the output of each single–product firm in each market.

The system to be solved becomes

$$\begin{pmatrix} \kappa+1 & \nu \\ \kappa & \nu+1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} M-\beta \\ M \end{pmatrix} \tag{15}$$

Solution of (15) gives equilibrium[7] quantities. The output of a multiproduct firm in each market is given by

$$x = \frac{M - \beta(1+\nu)}{1+\kappa+\nu} \tag{16}$$

The output of each single–product firm in each market is given by

$$y = \frac{M + \beta\kappa}{1+\kappa+\nu} \tag{17}$$

The industry structure $S = (\kappa, \nu)$ is non-trivial, if all firms in it have positive sales in the markets in which they participate. From equations (16), and (17) we see that this condition is satisfied for $M/(1+\nu) > \beta > -M/\kappa$.

The total equilibrium quantity of each product is given by

$$Q = Q_1 = Q_2 = \frac{M(\kappa+\nu) - \beta\kappa}{1+\kappa+\nu} \tag{18}$$

From equations (16) and (17) I obtain the following result:

[6] I assume *symmetric industry configurations*, in order to derive the analytical expressions of equilibrium strategies. **Tables 1** and **2** provide the results of simulations with *asymmetric industry configurations.*

[7] Equilibrium is locally strictly stable for all non negative κ, and ν. Under the same condition, stability of equilibrium in each market is independent of the equilibrium in the other market. Note that $(\kappa+1)(\nu+1) > \kappa\nu$.

Result 1 *In the presence of cost substitutability (complementarity), the output of a multiproduct firm in any of the two markets is lower (higher) than the output of a single–product firm in the same market.*

From equation (18) I obtain:

Result 2 *The total output of each of the two products is lower (higher) the higher the number of multiproduct firms and the higher the degree of cost substitutability (complementarity) between the two products.*

The assumption of linear, downward–sloping demand functions implies that higher quantities lead to lower prices and a higher consumers' surplus, while the deadweight welfare loss decreases. I consider the total output as a measure of social welfare. The implication of Results 1 and 2 is that, in the presence of cost complementarity (substitutability), a large (small) number of multiproduct firms leads to high (low) levels of social welfare. The effect of multiproduct technologies on welfare is greater, the greater the degree of cost complementarity or substitutability between the two products. Joint production of cost complements leads to high levels of efficiency and social welfare, whereas the opposite holds for joint production of cost substitutes.

Equilibrium profits of a single–product firm n are given by

$$\Pi_n = \left(\frac{M + \beta\kappa}{1 + \kappa + \nu}\right)^2 - F_i \tag{19}$$

where i denotes the market in which the firm operates.

The profits of a multiproduct firm k are given by

$$\Pi_k = 2\left(\frac{M - \beta(1 + \nu)}{1 + \kappa + \nu}\right)^2 - F_{12} \tag{20}$$

In an industry whose structure is given by $S = (\kappa, \nu)$, the profits from multiproduct activity are superadditive with respect to the profits from single–product activity if $\Pi_k > 2\Pi_n$. From equations (19) and (20) we get the following proposition:

Proposition 1 *In the presence of cost complementarity and weak subadditivity of fixed costs, the profits of a multiproduct firm are superadditive with respect to the profits from multiproduct activity for all values of the parameter β. In the absence of production relations between the two products the profits of a multiproduct firm are superadditive (weakly superadditive) if the fixed costs of* *joint production are subadditive (weakly subadditive) with respect to the fixed costs of the 'stand alone' production of the two products. In the presence of cost substitutability, there is a minimum level of subadditivity of fixed costs beyond which the profits of a multiproduct firm are superadditive with respect to the profits from single–product activity.*

Proof: From equations (19) and (20) I get that $\Pi_k > \Pi_l + \Pi_m$ holds if $F_1 + F_2 - F_{12} > \beta R_{S=(\kappa,\nu)}$, where

$$R = 2\frac{2M + \beta(\kappa - \nu - 1)}{1 + \kappa + \nu}$$

Remember that $M/(1+\nu)\} > \beta > -M/\kappa$ is a necessary condition for the industry configuration $S = (\kappa, \nu)$ to be non trivial. Then the expression $2M + \beta(\kappa - \nu - 1)$ is positive. Remember also that the fixed costs of a multiproduct firm were assumed to be subadditive with respect to the fixed costs of the 'stand alone' production of the two goods. Therefore, $F_1 + F_2 - F_{12} \geq 0$. Then for all negative values of the parameter β it is true that $F_1 + F_2 - F_{12} \geq 0 > \beta R_{S=(\kappa,\nu)}$.

For $\beta = 0$ the profits of a multiproduct firm are superadditive as compared to the profits from single–product activity if $F_1 + F_2 - F_{12} > 0$. If $F_1 + F_2 = F_{12}$ then the profits of a multiproduct firm are equal to the sum of profits of two single–product firms, one from each market.

Finally, for $\beta > 0$, for each industry configuration $S = (\kappa, \nu)$ there is a minimum level of subadditivity of fixed costs, given by $\beta R_{S=(\kappa,\nu)}$, for which a multiproduct firm earns more profits than the sum of profits of two of its single–product rivals (one in each market). □

The implication of proposition 1 is that the forces of the market reward efficient firms with higher profits in each one of the markets in which they participate. Profitability of multiproduct activity requires either cost complementarity or subadditivity of fixed costs. If multiproduct firms are free to choose between multiproduct single–plant activity and single–product multi–plant activity, the forces of the market will lead the industry along efficient configurations. Following Result 2, cost substitutability results in lower levels of social welfare. Therefore, if efficiencies due to subadditivity of fixed costs offset inefficiencies due to cost substitutability, multiproduct activity is profitable, but leads to lower levels of social welfare. In that case, the forces of the market guarantee high levels of efficiency but not high levels of social welfare.

I compare now the profits of a multiproduct firm to the profits of any one of its single–product rivals. Multiproduct activity is profitable as long as the profits of a multiproduct firm exceed the profits of any single–product firm in any of the two markets. I assume that the fixed costs of a single–product manufacturer of product 1 are equal to the fixed costs of a single–product manufacturer of product 2. From equations (19) and (20) I obtain the condition for which $\Pi_k > \Pi_n$.

Result 3 *In an industry whose structure is given by $S = (\kappa, \nu)$, multiproduct activity is more profitable than single product activity if*

$$\frac{[(\sqrt{2}-1)M - \beta(\sqrt{2}(1+\nu)+\kappa)][(\sqrt{2}+1)M - \beta(\sqrt{2}(1+\nu)-\kappa)]}{(1+\kappa+\nu)^2} > F_{12} - F \tag{21}$$

where $F = F_1 = F_2$.

I impose further assumptions on the parameters of the model to discuss an interesting implication of Result 3. I eliminate subadditivity of fixed costs, assuming $F = F_{12}$. Due to inequality (21), multiproduct activity is more profitable than single–product activity if

$\beta \leq 0$ and $\sqrt{2}(1+\nu) \geq \kappa$

or if $\beta > 0$ and $\sqrt{2}(1+\nu) \geq \kappa$ hold simultaneously with

$$\frac{(\sqrt{2}-1)M}{\sqrt{2}(1+\nu)+\kappa} > \beta \tag{22}$$

and

$$\frac{(\sqrt{2}+1)M}{\sqrt{2}(1+\nu)-\kappa} > \beta \tag{23}$$

Note that

$$\frac{(\sqrt{2}-1)M}{\sqrt{2}(1+\nu)+\kappa} < \frac{(\sqrt{2}+1)M}{\sqrt{2}(1+\nu)-\kappa} \tag{24}$$

Then, inequalities (22) and (23) hold simultaneously for all positive values of the parameter β for which inequality (22) holds. In that case, multiproduct activity is more profitable, although the joint production of cost substitutes is not only socially undesirable, but also inefficient.

I have eliminated the possibility of economies of scope due to subadditivity of fixed costs. In this way, the range of diseconomies of scope, under which multiproduct activity is profitable, is expressed in a range of positive values of the parameter β (the range given by inequality (22)). I have shown that in this range of values of the parameter β, multiproduct firms earn higher profits than single–product firms, although multiproduct activity is inefficient and leads to lower levels of social welfare. The range of positive values which satisfy inequality (22) is broader for small numbers of firms in the industry and for high values of the parameter M.

This consequence of Result 3 holds under a number of restrictive assumptions. **Table 1** presents similar results that were obtained from simulations with *symmetric* and *asymmetric industry configurations*. Multiproduct activity can be more profitable even in *asymmetric industry configurations* and in the presence of cost substitutability between the two products. A multiproduct firm earns higher profits than any of its single–product rivals unless the degree of substitutability between the two products is very high.[8] An interesting implication of the comparison between configurations $S = (1, 2, 3)$ and $S = (2, 2, 2)$ is that a single–product firm in market 2 in $S = (1, 2, 3)$ would be willing to enter into market 1 if it possessed the *know–how* for the production of the first product, unless β were too high.[9] Furthermore, a *potentially multiproduct* firm would prefer single–product activity only if β were too high.[10]

Therefore, configurations that contain more multiproduct firms may be internally stable even in the presence of inefficiencies of joint production. The forces of the market do not guarantee efficient industry configurations, unless the degree of substitutability between the two products is very high. The existence of subadditivity of the fixed costs of multiproduct firms, as compared to the sum of fixed costs of *stand–alone* production, would result in an even broader range of cost substitutability under which *potentially multiproduct firms* would prefer multiproduct activity than participation in only one of the two markets.

3 Multiproduct Industries and Long–Run Equilibrium Industry Configurations

In this section I extend the definition of the *long–run equilibrium number of firms* to its multiproduct analogous, the *long–run equilibrium industry configuration* (LREC).

> An industry configuration $S = (\kappa, \lambda, \mu)$ is a long–run equilibrium configuration (LREC) of a multiproduct industry if the numbers κ, λ and μ are such that all firms in the industry earn zero profits.

[8] With the parameter values assumed in the simulations presented in **Table 1**, multiproduct firms earn higher profits than any of their single–product rivals for all $\beta < 10$.

[9] In the example of **Table 1** if $\beta = 10$. Note that for all other values of β multiproduct activity is more profitable. Compare for example 441 (which are the profits of a single–product firm in market 2 in $S = (1, 2, 3)$, if $\beta = 5$) with 578 (which are the profits that the firm would earn if it entered into market 1).

[10] Compare the profits of a multiproduct firm in $S = (2, 2, 2)$ with the profits of a single-product firm in market 2 in $S = (1, 2, 3)$.

Table 1									
Short–run equilibrium quantities and profits									
$S=(1,2,3)$									
β	x_1	x_2	y_1	y_2	Q_1	Q_2	Π_k	Π_l	Π_m
0	25.00	20.00	25.00	20.00	75.00	80.00	1025	625	400
0.1	24.92	19.92	25.02	20.02	74.95	79.98	1018	626	401
1	24.25	19.20	25.25	20.20	74.75	79.80	957	637	408
5	21.25	16.00	26.25	21.00	73.75	79.00	709	689	441
10	17.50	12.00	27.50	22.00	72.50	78.00	455	756	484
$S=(2,2,2)$									
β	x_1	x_2	y_1	y_2	Q_1	Q_2	Π_k	Π_l	Π_m
0	20.00	20.00	20.00	20.00	80.00	80.00	800	400	400
0.1	19.94	19.94	20.04	20.04	79.96	79.96	795	401	401
1	19.40	19.40	20.40	20.40	79.60	79.60	752	416	416
5	17.00	17.00	22.00	22.00	78.00	78.00	578	484	484
10	14.00	14.00	24.00	24.00	76.00	76.00	392	576	576

$M=100$, $F_{12}=F_1=F_2=0$

I apply the results obtained in the previous section to study the properties of the long–run equilibrium of a multiproduct industry. I assume that a long–run equilibrium configuration is reached after a number of firms enter into the industry. Positive profits to a certain type of firms cause the number of firms of this type to increase. Negative profits to a ceratin type of firms cause the number of firms of this type to decrease.

I deal with the *integer problem* in two different ways. First, *fractions of firms* are permitted. In Baumol *et al.* (1982) this assumption is used to study the existence and structure of multiproduct competitive equilibria. The assumption becomes more plausible for large number of firms and in the case of industries in which firms are free to enter into the market at a smaller scale than the equilibrium output in a Cournot oligopoly. Unique or multiple long–run equilibria are obtained, depending on the degree of substitutability or complementarity between the two products and on the extent to which fixed costs of joint production are subadditive to the sum of fixed costs of 'stand alone' production. Then, I permit only an integer number of firms of each type. The definition of a *long–run equilibrium industry configuration* becomes slightly different.

An industry configuration $S_0=(\kappa_0,\lambda_0,\mu_0)$ is a long–run equilibrium configuration of a multiproduct industry, if the integer numbers κ_0, λ_0 and μ_0 are such that all firms in the industry earn non–negative profits, while the entry of any type of firm would yield negative profits.

This alternative approach to the *long–run equilibrium industry configuration* is more appropriate for small number of firms and industries in which firms are bound to enter at an indivisible scale of production which is equal to the equilibrium quantity of a Cournot oligopoly. The main implication is that firms earn positive profits in the long run. I find that there are broader conditions, compared with the case of *fractional long–run equilibrium*, under which there are multiple equilibria.

3.1 Fractional Long–Run Equilibrium Industry Configurations

I assume that the fixed costs of single–product firms are such that $F_1 = F_2 = F$. Following this assumption, a *long–run equilibrium industry configuration* contains equal numbers of single–product firms in each market.[11]

In the long run, firms will enter into the industry until the profit of each type of firm is zero. From equations (19) and (20) this holds if

$$\Pi_n = \left(\frac{M+\beta\kappa}{1+\kappa+\nu}\right)^2 - F = 0 \tag{25}$$

and

$$\Pi_k = 2\left(\frac{M-\beta(1+\nu)}{1+\kappa+\nu}\right)^2 - F_{12} = 0 \tag{26}$$

From equation (25) I get

$$\nu = \frac{M}{\sqrt{F}} - 1 - (1 - \frac{\beta}{\sqrt{F}})\kappa \tag{27}$$

where $M \geq F$, in order for the number of single–product firms to be non-negative and $\beta < F$, so that a simultaneous increase in the number of multi-product and single–product firms cannot be sustained in the long run. Furthermore, equation (27) gives us the upper bound on κ, implied by the requirement that ν is nonnegative, which is given by

$$\frac{\frac{M}{\sqrt{F}} - 1}{1 - \frac{\beta}{\sqrt{F}}} \geq \kappa$$

Substitution of ν from (27) into (26) gives

$$2(\sqrt{F} - \beta)^2 = F_{12} \tag{28}$$

[11] **Table 2** presents the results of simulations with *asymmetric long–run equilibrium industry configurations.*

Equation (28) is independent of the number of multiproduct firms κ. Then I reach the following proposition:

Proposition 2 *If the parameters F_{12}, F and β are such that (28) holds, all industry configurations $S = (\kappa, \nu)$, for $\kappa \geq 0$ and $\nu \geq 0$, for which equation (27) holds, are long–run equilibrium industry configurations.*

Equation (27) has an infinite number of solutions. Therefore, if (28) holds, there is an infinite number of long–run equilibrium configurations of the industry. A straightforward implication of proposition 2, and especially equation (27) is the following result:

Result 4 *In a multiproduct industry which has reached a LREC an increase (decrease) in the number of multiproduct firms can lead to another LREC only if it is followed by a decrease (increase) in the number of single–product firms.*

Equation (28) implies that if the fixed costs of joint production are subadditive to the sum of the fixed costs of 'stand alone' production multiple equilibria result only in the presence of cost substitutability. Therefore, multiproduct firms co–exist in the long run with single–product firms, if multiproduct activity leads to efficiency losses from cost substitutability and efficiency gains from subadditivity of fixed costs. In proposition 1, I obtained a similar result for the short run. The configurations given by equation (27) are equally possible to sustain in the long run. However, configurations that contain more manufacturers of cost substitutes lead to lower levels of social welfare (see Result 2).

Equation (28) holds also in the absence of production relation between the two products and if the fixed costs of joint production are equal to the sum of fixed costs of the 'stand alone' production. In that case, the different long–run equilibria result in identical levels of efficiency and social welfare (see Result 2).

In the general case, equation (28) does not hold. Then, the expression holds as an inequality. If

$$2(\sqrt{F} - \beta)^2 > F_{12} \tag{29}$$

the profits of a multiproduct firm are always positive for all industry configurations in which single–product firms earn zero profits. Then the entry of a multiproduct firms is always profitable. For equation (27) to hold, an increase in the number of multiproduct firms is followed by a decrease in the numbers of single–product firms. The unique equilibrium industry configurations contains only multiproduct firms. I substitute $\nu = 0$ into equation (28). I obtain the following proposition:

Proposition 3 *If inequality (29) holds, the long–run equilibrium configuration is $S = (\kappa, 0)$, where*

$$\kappa = \frac{\sqrt{2}(M - \beta)}{\sqrt{F_{12}}} - 1$$

Alternatively, the expression given by (28) holds as an inequality in the opposite direction.

$$2(\sqrt{F} - \beta)^2 < F_{12} \tag{30}$$

Then the profits of a multiproduct firm are always negative for all industry configurations in which each single–product firm earns zero profits. All multiproduct firms exit from the industry and the long–run equilibrium configuration contains only single–product firms. I set $\kappa = 0$. From equation (27) I obtain the following proposition:

Proposition 4 *If inequality (30) holds, the long–run equilibrium configuration is $S = (0, \nu)$, where*

$$\nu = \frac{M}{\sqrt{F}} - 1$$

Propositions 3 and 4 indicate that if there is a unique *long–run equilibrium configuration* high (low) fixed costs result in a low (high) number of firms. This follows the standard result obtained from the single–product approach. Proposition 3 shows that a unique *long–run equilibrium configuration* which consists of multiproduct firms contains less firms the higher the degree of production substitutability between the two products is. Therefore, the negative short–run effect of cost substitutability on equilibrium outputs, affects negatively the long–run number of multiproduct firms. Note that in the absence of multiproduct firms, the degree of production substitutability or complementarity does not affect the number of single–product firms.

Propositions 3 and 4 give us the conditions under which there is a unique *symmetric long–run equilibrium industry configuration.* In the presence of global economies of scope, the long–run equilibrium configuration contains only multiproduct firms, whereas in the presence of global diseconomies of scope the long–run industry configuration contains only single–product firms. Given that equation (28) is not satisfied under subadditivity of fixed costs and cost complementarity, multiple mixed equilibria are obtained only in the presence of local economies of scope, or in the absence of any economies or diseconomies of joint production. Proposition 3 indicates that if the fixed costs of joint production are sufficiently lower than the sum of the fixed costs of 'stand alone' production of the two products, the long–run equilibrium configuration contains only multi–product firms. In that case, multiproduct activity is profitable, efficient and sustainable in the long run. However, efficiency results from subadditivity of fixed costs. In the absence of fixed costs the long–run equilibrium configuration would contain only single–product firms and would result in higher levels of social welfare. Consider the case in which $\beta > 0$. Compare $S_1 = (\kappa, 0)$ and $S_2 = (0, \kappa)$ where κ is the number of firms given by proposition 3. The industry configuration S_2 leads to higher levels of social welfare than S_1. However, if inequality (29) holds, S_1 is the unique long–run equilibrium configuration. In that case, the long–run equilibrium configuration is a suboptimal outcome in terms of social welfare.

3.2 Long–Run Equilibrium Configurations with Integer Numbers of Firms

If the number of firms is small or the entry of a firm is possible only at a scale of production which is indivisible, the assumption of 'fractions of firms' becomes less plausible. An industry configuration $S_0 = (\kappa_0, \nu_0)$, where κ_0 and ν_0 are positive integer numbers, is a long-run equilibrium configuration, if each type of firm in it earns non–negative profits and the entry of any type of firm into the industry yields negative profits.

If κ_0 and ν_0 are integer numbers, the conditions for $S_0 = (\kappa_0, \nu_0)$ to be a long–run equilibrium industry configuration become

$$\Pi_n = \left(\frac{M + \beta\kappa_0}{1 + \kappa_0 + \nu_0} \right)^2 - F = \tau \tag{31}$$

and

$$\Pi_k = 2 \left(\frac{M - \beta(1 + \nu_0)}{1 + \kappa_0 + \nu_0} \right)^2 - F_{12} = \tau_{12} \tag{32}$$

where $\tau \geq 0$, and $\tau_{12} \geq 0$ give the net profits of each type of firm in the long run. If the fractional long–run equilibrium coincides with the long–run equilibrium with integer numbers, firms earn zero net profits. In the general case, in a long-run equilibrium configuration $S = (\kappa_0, \nu_0)$, each multiproduct firm earns positive net profits given by τ_{12} and each single–product firm earns τ. It must hold that

$$\tau < \Pi_n^{S=(\kappa_0,\nu_0)} - \Pi_n^{S=(\kappa_0,\nu_0+1)} \tag{33}$$

and

$$\tau_{12} < \Pi_k^{S=(\kappa_0,\nu_0)} - \Pi_k^{S=(\kappa_0+1,\nu_0)} \tag{34}$$

If one of (33), and (34) is not satisfied, then the entry of a certain type of firm is sustainable. In that case, $S = (\kappa_0, \nu_0)$ would not be a long–run equilibrium configuration. Applying equations (19), and (20), I find that inequalities

(33) and (34) hold for

$$\tau < t(M + \beta\kappa_0)^2 \tag{35}$$

and

$$\tau_{12} < 2t(M - \beta(1 + \nu_0))^2 \tag{36}$$

where

$$t = \frac{3 + 2\kappa_0 + 2\nu_0}{(1 + \kappa_0 + \nu_0)^2(2 + \kappa_0 + \nu_0)^2} \tag{37}$$

From equation (31) I obtain the condition that has to be satisfied by a long–run equilibrium configuration with integer numbers of firms:

$$\nu_0 = \frac{M}{\sqrt{F + \tau}} - 1 - (1 - \frac{\beta}{\sqrt{F + \tau}})\kappa_0 \tag{38}$$

where $M \geq \sqrt{F + \tau}$, in order for the number of single–product firms to be non–negative and $\beta < \sqrt{F + \tau}$, so that a simultaneous increase in the number of multiproduct and single–product firms cannot be sustained in the long run. Furthermore,

$$\frac{\frac{M}{\sqrt{F+\tau}} - 1}{1 - \frac{\beta}{F+\tau}} \geq \kappa$$

denotes the upper bound on κ implied by the requirement that the number of single–product firms is nonnegative.

Substitution of ν_0 from (38) into (32) gives

$$2(\sqrt{F + \tau} - \beta)^2 = F_{12} + \tau_{12} \tag{39}$$

which is similar to the condition given by (28) for the 'fractional' long–run equilibrium. According to (33) and (34), τ_{12}, and τ are bounded by quantities which are decreasing functions of κ_0 and ν_0. Therefore, the condition given by (39), unlike (28), depends on the number of firms.

Long–run equilibrium configurations should satisfy simultaneously (38) and (39). Then, I get the following proposition:

Proposition 5 *The set of industry configurations $\Omega = \{S_1, S_2, ...S_n\}$ with integer numbers of firms is the set of long–run equilibrium configurations, if all configurations in it satisfy (38) and (39).*

If there is more than one industry configuration for which (39) holds, then we have *multiple mixed equilibria*. Each one of them contains multiproduct and single–product firms. Any of the multiple mixed long–run equilibria can be reached and sustained in the long run. From equation (39), I obtain the following result:

Result 5 *If* $2F \geq F_{12}$, $\beta \geq 0$ *and the industry configuration* $S = (\kappa_0, \nu_0)$ *is a mixed long–run equilibrium configuration, then* $2\tau < \tau_{12}$ *implies a lower critical level of subadditivity of fixed costs, beyond which multiproduct firms exist in the presence of cost substitutability.*

Result 5 is obtained directly from propositions 1 and 5 and indicates that inefficient and socially undesirable industry configuration can be sustained in the long run. Following result 2, configurations with more multiproduct firms are socially desirable in the presence of cost complementarity and socially undesirable in the presence of cost substitutability. The intuitive interpretation of proposition 5 and result 5 is that if only integer number of firms are permitted, there is a broader range of conditions, as compared to the case of 'fractional' equilibrium, under which mixed equilibria exist. Suboptimal outcomes with respect to efficiency and social welfare can be sustained in the long run.

If no industry configuration satisfies (39), then the expression in (39) holds as an inequality.

Proposition 6 *If for all industry configurations it is true that*

$$2(\sqrt{F+\tau} - \beta)^2 > F_{12} + \tau_{12} \tag{40}$$

there is a unique long–run equilibrium configuration $S = (\kappa_0, 0)$ *where* κ_0 *is the integer part of*

$$\kappa = \frac{\sqrt{2}(M-\beta)}{\sqrt{F_{12}}} - 1$$

Proposition 7 *If for all industry configurations it is true that*

$$2(\sqrt{F+\tau} - \beta)^2 < F_{12} + \tau_{12} \tag{41}$$

there is a unique long–run equilibrium configuration $S = (0, \nu_0)$ *where* ν_0 *is the integer part of*

$$\nu = \frac{M}{\sqrt{F}} - 1$$

The last two propositions follow directly from propositions 3 and 4.

We have seen that the long–run equilibrium configuration may contain multiproduct and single–product firms. 'Fractional' long equilibrium results in efficient industry configurations, but not necessarily optimal outcomes in terms of social welfare. Multiple and mixed long–run equilibria are obtained under a broader range of conditions if the industry configurations are assumed to contain integer numbers of firms. The outcome may be inefficient and suboptimal in terms of social welfare.

Table 2									
Long–run equilibrium configurations									
S_0	$x_{1,0}$	$x_{2,0}$	$y_{1,0}$	$y_{2,0}$	$Q_{1,0}$	$Q_{2,0}$	Π_{k0}	Π_{l0}	Π_{m0}
(0, 5, 6)	16.16	13.78	16.66	14.28	83.33	85.71	451	277	204
(1, 4, 5)	16.25	13.85	16.75	14.35	83.25	85.64	456	280	206
(2, 3, 4)	16.33	13.92	16.83	14.42	83.16	85.57	460	283	208
(3, 2, 3)	16.41	14.00	16.91	14.50	83.08	85.50	465	286	210
(4, 1, 2)	16.50	14.07	17.00	14.57	83.00	85.42	470	289	212
(5, 0, 1)	16.58	14.14	-	14.64	82.91	85.32	475	-	214

$\beta = 0.5, M = 100, F_{12} = 400, F_1 = 250, F_2 = 200$

Note that profits are **not** net of fixed costs.

Table 2 presents the results of simulations obtained under the assumption that $F_1 \neq F_2$. *Asymmetric long–run equilibrium industry configurations* contain more single–product firms of the type that operates with lower fixed costs.

All configurations presented in **Table 2** satisfy the necessary condition for a *long–run equilibrium configuration.* That is, firms in each configuration earn nonnegative net profits, while a potential entrant of any of the three types would earn negative profits. Note that *internal industry dynamics* may still take place and lead from one LREC to another. Result 4 indicates that increases in the number of multiproduct firms are sustainable in the long run if they are followed by decreases in the numbers of single–product firms. An interesting implication of this result is that after an industry has reached equilibrium, industry dynamics may still occur and lead the industry along different long–run equilibria. The policy–maker can use this rule to restructure the industry without causing firms' profits to fall below the critical level of survival. A special case of industry dynamics that increases the number of multiproduct firms and decreases the number of multiproduct firms is an inter–market merger. An inter–market merger is a merger of two single–product firms from two different markets into a multiproduct firm.[12] A split–up of a multiproduct firm into two single–product firms has the opposite effect on the number of firms: the number of multiproduct firms decreases and the number of single-product firms increases.[13]

[12]In Georgantzis (1992), special attention is given to the effect of inter–market mergers on potential competition.

[13]Note that in the example of **Table 2** such dynamics can move the industry from one of the *long–run equilibrium configurations* to another. In this case, intermarket mergers are profitable for the merging firms although production becomes less efficient and total output falls.

Note also that the industry configuration $S = (6, 0, 0)$ satisfies the necessary condition for a long–run equilibrium configuration, but is internally unstable, given that a potentially multiproduct firm in it earns higher long–run profits from single-product activity in market 2. The LREC that emerges is given by $S = (5, 0, 1)$.

4 Conclusions

I have studied the effect of production complementarity and substitutability, on the equilibria in two markets which are served by a number of multiproduct firms and a number of single–product firms in each market. Production complementarity gives a technological advantage to a multiproduct manufacturer, while the opposite is true for production substitutability. Production substitutability has a negative effect on output levels. Therefore, joint production of cost substitutes has a negative impact on social welfare. In the presence of cost substitutability, savings due to subadditivity of fixed costs of multiproduct firms as compared to the sum of the fixed costs of 'stand–alone' production, makes multiproduct activity profitable but not socially desirable.

Production complementarity or substitutability and subadditivity of fixed costs determine the industry structure in the long run. Multiple mixed equilibria result under a broader range of conditions, if the industry configurations are assumed to contain integer numbers of firms than if 'fractions' of firms are permitted. 'Fractional' equilibria guarantee efficiency but not optimality in terms of consumer welfare. If industry configurations are assumed to consist of integer numbers of firms, socially suboptimal and inefficient outcomes can be sustained in the long run.

In the case of *multiple long–run equilibrium configurations*, 'moving' from one of them to another implies that increases in the number of multiproduct firms are followed by decreases in the number of single–product firms and vice versa. The implication of this observation is that the merger of two single–product manufacturers into a multiproduct firm may prove a useful tool for restructuring a multiproduct industry without affecting significantly incumbents' long–run profits.

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